# Latent Variable Hybrids: Overview Of Old And New Models

Bengt Muthén  
UCLA  
bmuthen@ucla.edu

Presentation at the University of Maryland CILVR conference  
"Mixture Models in Latent Variable Research",  
May 18-19, 2006

## Overview

<table>
<thead>
<tr>
<th></th>
<th>Continuous Latent Variables</th>
<th>Categorical Latent Variables</th>
<th>Hybrids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-Sectional Models</td>
<td>Factor analysis, SEM</td>
<td>Regression mixture analysis, Latent class analysis</td>
<td>Factor mixture analysis</td>
</tr>
<tr>
<td>Longitudinal Models</td>
<td>Growth analysis (random effects)</td>
<td>Latent transition analysis, Latent class growth analysis</td>
<td>Growth mixture analysis</td>
</tr>
</tbody>
</table>
Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

Measurement Invariance
- Yes
- No

Factor Analysis Emphasis
- Yes
- No

Cluster Analysis and Factor Analysis Emphasis

Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

Measurement Invariance
- Yes
- No

Factor Analysis Emphasis
- Yes
- No

Parametric Factor Distribution
- Yes
- No

Mixture Factor Analysis

factor
Overview Of Hybrids: Measurement Invariance, Parametric Factor Distribution

- Cross-sectional examples:

Overview Of Hybrids: Measurement Invariance, Parametric Factor Distribution

- Longitudinal examples:
  - Growth mixture modeling of trajectory classes (Verbeke & LeSaffre, 1996; Muthen & Shedden, 1999)
  - Intervention effects varying across trajectory classes (Muthen et al, 2002)
  - Regime (latent class) switching (Dolan, Schmittman, Lubke, Neale, 2005)
Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

- Measurement Invariance
  - Yes
  - No

- Factor Analysis Emphasis
  - Yes
  - No

- Parametric Factor Distribution
  - Yes
  - No

- Mixture Factor Analysis
  - Yes
  - No

- Non-Parametric FA

Overview Of Hybrids: Non-Parametric Factor Distribution

- Cross-sectional examples:
  - Latent class factor analysis for DSM-V (Muthen & Asparouhov, 2006)
  - IRT (?)

- Longitudinal examples:
  - Binary growth; non-normal random effects (Aitkin, 1999)
  - Trajectory groups for criminal offenders (Nagin & Land, 1993)
Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

- **Measurement Invariance**: Yes / No
  - Factor Analysis Emphasis: Yes / No
    - Parametric Factor Distribution: Yes / No
      - Mixture Factor Analysis: Yes / Non-Parametric FA

### Cross-sectional examples:
- Factor mixture analysis in psychometrics (Blafield, 1980; Yung, 1997)
- Structural equation mixtures; market segmentation (Jedidi, Jagpal & DeSarbo, 1997)
- IRT mixtures; solution strategies, guessing, levels of difficulty - Saltus (Yamamoto, 1987; Mislevy & Verhelst, 1990; Mislevy & Wilson, 1996; Wilson, de Boeck, Acton, 2005)


Overview Of Hybrids: Factor Mixture Analysis

- Cross-sectional examples:
  - Factor mixture analysis in psychometrics (Blafield, 1980; Yung, 1997)
  - Structural equation mixtures; market segmentation (Jedidi, Jagpal & DeSarbo, 1997)
  - IRT mixtures; solution strategies, guessing, levels of difficulty - Saltus (Yamamoto, 1987; Mislevy & Verhelst, 1990; Mislevy & Wilson, 1996; Wilson, de Boeck, Acton, 2005)
Overview Of Hybrids: Factor Mixture Analysis (Continued)

• Cross-sectional examples (continued):
  • Factor mixture analyzers; continuous micro-array expression data (McLachlan, Do & Ambroise, 2004)
  • Factor mixture modeling; binary diagnostic criteria; genetics for twins, siblings (Muthen, Asparouhov, Rebollo, 2006)
  • Classic normal finite mixtures; Fisher’s Iris data
  • Non-parametric factor mixture analysis (?)

• Longitudinal examples:
  • Factor mixture latent transition analysis (Muthen, 2006)

Unclassified Contributors

• Dayton, Meehl, Vermunt

• Suggestions welcome
Model Testing Issues

NLSY 1989: Latent Class Analysis Of DSM-III-R Alcohol Dependence Criteria (n = 8313)


<table>
<thead>
<tr>
<th>Latent Classes</th>
<th>Two-class solution</th>
<th>Three-class solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Prevalence</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>DSM-III-R Criterion</td>
<td>Conditional Probability of Fulfilling a Criterion</td>
<td></td>
</tr>
<tr>
<td>Withdrawal</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>Tolerance</td>
<td>0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>Larger</td>
<td>0.15</td>
<td>0.96</td>
</tr>
<tr>
<td>Cut down</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>Time spent</td>
<td>0.00</td>
<td>0.19</td>
</tr>
<tr>
<td>Major role-Hazard</td>
<td>0.03</td>
<td>0.83</td>
</tr>
<tr>
<td>Give up</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Relief</td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td>Continue</td>
<td>0.00</td>
<td>0.24</td>
</tr>
</tbody>
</table>

1Likelihood ratio chi-square fit = 1779 with 492 degrees of freedom
2Likelihood ratio chi-square fit = 448 with 482 degrees of freedom
LCA, 3 classes: \( \log L = -14,139 \), 29 parameters, BIC = 28,539
FA, 2 factors: \( \log L = -14,083 \), 26 parameters, BIC = 28,401
FMA 2 classes, 1 factor, loadings invariant:
\( \log L = -14,054 \), 29 parameters, BIC = 28,370

Deciding On The Number Of Classes:
Bootstrapped LRT

- Nylund, Muthen and Asparouhov (2006) simulation study
- BLRT has better Type I error than NCS and LMR
- BLRT finds the right number of classes better than BIC and LMR
Latent class analysis with categorical outcomes

<table>
<thead>
<tr>
<th>Model</th>
<th>BIC</th>
<th>LMR</th>
<th>BLRT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Classes</td>
<td>Classes</td>
<td>Classes</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>10-Item (Complex Structure) n=200</td>
<td>92</td>
<td><strong>8</strong></td>
<td>0</td>
</tr>
<tr>
<td>n=500</td>
<td>24</td>
<td><strong>76</strong></td>
<td>0</td>
</tr>
<tr>
<td>n=1000</td>
<td>0</td>
<td><strong>100</strong></td>
<td>0</td>
</tr>
</tbody>
</table>

Factor (IRT) Mixture Example: The Latent Structure Of ADHD

- UCLA clinical sample of 425 males ages 5-18, all with ADHD diagnosis
- Subjects assessed by clinicians:
  1) direct interview with child (> 7 years),
  2) interview with mother about child
- KSADS: Nine inattentiveness items, nine hyperactivity items; dichotomously scored
- Families with at least 2 ADHD affected children
- Parent data, candidate gene data on sib pairs
### The Latent Structure Of ADHD (Continued)

<table>
<thead>
<tr>
<th>Inattentiveness Items</th>
<th>Hyperactivity Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Difficulty sustaining attention on tasks/play’</td>
<td>‘Difficulty remaining seated’</td>
</tr>
<tr>
<td>‘Easily distracted’</td>
<td>‘Fidgets’</td>
</tr>
<tr>
<td>‘Makes a lot of careless mistakes’</td>
<td>‘Runs or climbs excessively’</td>
</tr>
<tr>
<td>‘Doesn’t listen’</td>
<td>‘Difficulty playing quietly’</td>
</tr>
<tr>
<td>‘Difficulty following instructions’</td>
<td>‘Blurs out answers’</td>
</tr>
<tr>
<td>‘Difficulty organizing tasks’</td>
<td>‘Difficulty waiting turn’</td>
</tr>
<tr>
<td>‘Dislikes/avoids tasks’</td>
<td>‘Interrupts or intrudes’</td>
</tr>
<tr>
<td>‘Loses things’</td>
<td>‘Talks excessively’</td>
</tr>
<tr>
<td>‘Forgetful in daily activities’</td>
<td>‘Driven by motor’</td>
</tr>
</tbody>
</table>

### The Latent Structure Of ADHD: Model Fit Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Likelihood</th>
<th># Parameters</th>
<th>BIC</th>
<th>BLRT p value for k-1 classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCA – 2c</td>
<td>-3650</td>
<td>37</td>
<td>7523</td>
<td>0.</td>
</tr>
<tr>
<td><strong>LCA – 3c</strong></td>
<td><strong>-3545</strong></td>
<td><strong>56</strong></td>
<td><strong>7430</strong></td>
<td><strong>0.</strong></td>
</tr>
<tr>
<td>LCA – 4c</td>
<td>-3499</td>
<td>75</td>
<td>7452</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 5c</td>
<td>-3464</td>
<td>94</td>
<td>7496</td>
<td>0.</td>
</tr>
<tr>
<td><strong>LCA – 6c</strong></td>
<td><strong>-3431</strong></td>
<td><strong>113</strong></td>
<td><strong>7547</strong></td>
<td><strong>0.</strong></td>
</tr>
<tr>
<td>LCA – 7c</td>
<td>-3413</td>
<td>132</td>
<td>7625</td>
<td>0.27</td>
</tr>
</tbody>
</table>

LCA-3c is best by BIC and LCA-6c is best by BLRT
Three-Class And Six-Class LCA Item Profiles

The Latent Structure Of ADHD: Model Fit Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Likelihood</th>
<th># Parameters</th>
<th>BIC</th>
<th>BLRT p value for k-1 classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCA – 2c</td>
<td>-3650</td>
<td>37</td>
<td>7523</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 3c</td>
<td>-3545</td>
<td>56</td>
<td>7430</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 4c</td>
<td>-3499</td>
<td>75</td>
<td>7452</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 5c</td>
<td>-3464</td>
<td>94</td>
<td>7496</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 6c</td>
<td>-3431</td>
<td>113</td>
<td>7547</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 7c</td>
<td>-3413</td>
<td>132</td>
<td>7625</td>
<td>0.27</td>
</tr>
<tr>
<td>EFA – 2f</td>
<td>-3505</td>
<td>53</td>
<td>7331</td>
<td></td>
</tr>
</tbody>
</table>

The EFA model is better than LCA - 3c, but no classification of individuals is obtained.
The Latent Structure Of ADHD: Model Fit Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Likelihood</th>
<th># Parameters</th>
<th>BIC</th>
<th>BLRT p value for k-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCA – 2c</td>
<td>-3650</td>
<td>37</td>
<td>7523</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 3c</td>
<td>-3545</td>
<td>56</td>
<td>7430</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 4c</td>
<td>-3499</td>
<td>75</td>
<td>7452</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 5c</td>
<td>-3464</td>
<td>94</td>
<td>7496</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 6c</td>
<td>-3431</td>
<td>113</td>
<td>7547</td>
<td>0.</td>
</tr>
<tr>
<td>LCA – 7c</td>
<td>-3413</td>
<td>132</td>
<td>7625</td>
<td>0.27</td>
</tr>
<tr>
<td>EFA – 2f</td>
<td>-3505</td>
<td>53</td>
<td>7331</td>
<td></td>
</tr>
<tr>
<td>FMA – 2c, 2f</td>
<td>-3461</td>
<td>59</td>
<td>7280</td>
<td></td>
</tr>
<tr>
<td>FMA – 2c, 2f</td>
<td>-3432</td>
<td>75</td>
<td>7318</td>
<td>$\chi^2$-diff (16) = 58 p &lt; 0.0123</td>
</tr>
</tbody>
</table>

Three-Class LCA And Two-Class, Two-Factor FMA Item Profiles

![Graphs showing item profiles for different models](image)
Factor Mixture Modeling

Categorical outcomes plus continuous-normal latent variables have the statistical and computational disadvantage of

- normality assumption
- heavy computations due to numerical integration

Non-parametric latent variable distribution avoids the normality assumption and at the same time the computational disadvantage!

Non-Parametric Estimation Of The Random Effect Distribution Using Mixtures

Estimated weights and points (class probabilities and class means)
Longitudinal Analysis

- Factor analysis generalizes to random effects repeated measures (growth) analysis
- Latent class analysis generalizes to latent transition analysis
- Factor mixture analysis generalizes to growth mixture modeling and generalized latent transition analysis

Growth Mixture Modeling: Shapes of Growth Curves
A Clinical Trial
Of Depression Medication:
Two-Class Growth Mixture Modeling

Example: Mixed-Effects Regression Models For
Studying The Natural History Of Prostate Disease

Source: Pearson, Morrell, Landis and Carter (1994), Statistics in Medicine

Figure 2. Longitudinal PSA curves estimated from the linear mixed-effects model for the group average (thick solid line) and for each individual in the study (thin solid lines)
Growth Mixture Modeling
Of Developmental Pathways

Multilevel Growth Mixture Modeling
Growth Mixture Modeling: LSAY Math Achievement Trajectory Classes And The Prediction Of High School Dropout

Poor Development: 20%
Moderate Development: 28%
Good Development: 52%

Dropout: 69% 8% 1%

female hispanic black mother’s ed. home res. expectations drop thoughts arrested expelled

math7 math8 math9 math10

hsdrop

Muthen (2004)
Two-Part Growth Modeling Of Frequency Of Heavy Drinking Ages 18 – 25

Olsen and Schafer (2001)
Two-Part Modeling Extensions In Mplus

- Growth modeling
  - Distal outcome
  - Parallel processes
  - Trajectory classes (mixtures)
  - Multilevel
- Factor analysis
  - Mixtures
    - Latent classes for binary and continuous parts may be incorrectly picked up as additional factors in conventional analysis
  - Multilevel

Growth Modeling Paradigms:
Debate In Criminology And Annals AAPSS

Growth Mixtures
Random Effects  Muthen  Latent Classes
Raudenbush  Mplus  Nagin
HLM

troubled waters

Sampson & Laub
Growth Mixture Modeling Versus Latent Class Growth Analysis

**GMM**

Outcome

![Growth Mixture Modeling](image1)

**LCGA**

Outcome

![Latent Class Growth Analysis](image2)

Muthen

Nagin

Substantive And Non-Parametric Latent Classes

- Nagin’s inconsistency: Latent classes used as
  - Substantively distinct and meaningful subgroups
  - Non-parametric representation of the latent variable distribution

- Resolution: Combine substantive and non-parametric latent classes
  - Non-parametrically described variation within substantive themes
  - Easy to set up in Mplus using two latent class variables

3 non-parametric classes within each of 2 substantive classes
Transition Probabilities

<table>
<thead>
<tr>
<th>Mover Class (c=1)</th>
<th>Stayer Class (c=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>c1</td>
<td>1</td>
</tr>
<tr>
<td>c2</td>
<td>2</td>
</tr>
</tbody>
</table>

- Mover Class (c=1): u11, u12
- Stayer Class (c=2): u21, u22

Latent Transition Analysis

Time 1: u11, u12
Time 2: u21, u22

Latent Transition Analysis

Time 1: u11 to u1p
Time 2: u21 to u2p
1,137 first-grade students in Baltimore public schools

9 items: Stubborn, Break rules, Break things, Yells at others, Takes others property, Fights, Lies, Teases classmates, Talks back to adults

Skewed, 6-category items; dichotomized (almost never vs other)

Two time points: Fall and Spring of Grade 1

For each time point, a 2-class, 1-factor FMA was found best fitting

<table>
<thead>
<tr>
<th>Model</th>
<th>Loglikelihood</th>
<th># parameters</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional LTA</td>
<td>-8,649</td>
<td>21</td>
<td>17,445</td>
</tr>
<tr>
<td>FMA LTA factors related across time</td>
<td>-8,012</td>
<td>40</td>
<td>16,306</td>
</tr>
</tbody>
</table>
Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom (Continued)

Estimated Latent Transition Probabilities, Fall to Spring

<table>
<thead>
<tr>
<th></th>
<th>Conventional LTA</th>
<th>FMA-LTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Low</td>
<td>0.93</td>
<td>0.07</td>
</tr>
<tr>
<td>High</td>
<td>0.17</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Two-Level Latent Transition Analysis

\[ u_1 \ldots u_p \quad u_2 \ldots u_{2p} \]

Within

Between

\[ c_1 \quad c_2 \]

\[ c_1#1 \quad c_2#2 \]
Visit www.statmodel.com to check out

- Web videos of courses
- Recent papers
- Version 4 User’s Guide
- Mplus Discussion
- Short course announcements