Multilevel Mixture Models

Tihomir Asparouhov
Muthen & Muthen

Bengt Muthen
UCLA
Motivation

- Multilevel Models
- Mixture Models
- Structural Equation Models
- Mplus 3, Mplus 4.1, Comprehensive Modeling Framework
Topics

• The Basic Two-level Mixture Model

• Two-level Latent Transition Analysis

• Between Level Class Variables

• Grade of Membership Models
The Basic Two-level Mixture Model

$y_{ij}^*$ - observed dependent variables

$x_{ij}$ - within level covariates

$\eta_{ij}$ - continuous latent variables

$C_{ij}$ - categorical latent variables

$\eta_j$ - between latent variable (random effects)

$x_j$ - between level covariates
The Basic Two-level Mixture Model

\[ y_{ij}^* | C_{ij} = c = \nu_{cj} + \Lambda_{cj} \eta_{ij} + \varepsilon_{ij} \]

\[ \eta_{ij} | C_{ij} = c = \mu_{cj} + \Gamma_{cj} \eta_{ij} + B_{cj} x_{ij} + \xi_{ij} \]

\[ P(C_{ij} = c) = \frac{\exp(\alpha_{cj} + \beta_{cj} x_{ij})}{\sum_c \exp(\alpha_{cj} + \beta_{cj} x_{ij})}. \]

Within

\[ \eta_j = \mu + \Gamma \eta_j + B x_j + \xi_j. \]
Two-level Latent Transition Analysis

Multiple latent class variable $C_1, \ldots, C_T$.

Recursive System of Logit Models.

$$P(C_1 = 1) = \frac{\exp(\alpha_1j)}{\exp(\alpha_1j) + 1}$$

$$P(C_2 = 1|C_1) = \frac{\exp(\alpha_2j + \gamma I(C_1))}{\exp(\alpha_2j + \gamma I(C_1)) + 1}$$

$I(C_1)$ - indicator variable for $C_1 = 1$
Two-level Latent Transition Analysis continued

- Random Effects: $\alpha_1$ and $\alpha_2$.

- Within Level Logistic Regression: $C_1 \rightarrow C_2$

- Between Level Linear Regression: $\alpha_1 \rightarrow \alpha_2$
Two-level Latent Transition Analysis Example

• Baltimore study of aggressive and disruptive behavior in the classroom. TOCA instruments on binary scale. The substantive research question comes from Nick Ialongo at Johns Hopkins U.

• 2 class model for first grade fall ($C_1$) and spring ($C_2$).

• Disruptive class is 46% in the fall and 52% in the spring.

• Transition probabilities: $P(C_2 = 1|C_1 = 2) = 18\%$, $P(C_2 = 2|C_1 = 1) = 7\%$
## Two-level Latent Transition Analysis Example continued

<table>
<thead>
<tr>
<th>class</th>
<th>$C_t = 1$</th>
<th>$C_t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stubborn</td>
<td>0.92</td>
<td>0.36</td>
</tr>
<tr>
<td>Break Rules</td>
<td>0.96</td>
<td>0.29</td>
</tr>
<tr>
<td>Harm Others</td>
<td>0.73</td>
<td>0.03</td>
</tr>
<tr>
<td>Break Things</td>
<td>0.59</td>
<td>0.03</td>
</tr>
<tr>
<td>Yells at Others</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>Take Others’ Property</td>
<td>0.78</td>
<td>0.07</td>
</tr>
<tr>
<td>Fights</td>
<td>0.73</td>
<td>0.08</td>
</tr>
<tr>
<td>Lies</td>
<td>0.81</td>
<td>0.10</td>
</tr>
<tr>
<td>Tease Classmates</td>
<td>0.90</td>
<td>0.24</td>
</tr>
<tr>
<td>Trouble Accepting Authority</td>
<td>0.78</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Two-level Latent Transition Analysis Example continued
Two-level Latent Transition Analysis Example continued

- \( \alpha_1 \rightarrow C_1 \quad R^2 = 39\% \)

- \( \alpha_1, \alpha_2, C_1 \rightarrow C_2 \quad R^2 = 65\% \)
  - \( \alpha_1 \rightarrow C_2 \quad 35\% \)
  - residual \( C_1 \rightarrow C_2 \quad 25\% \) (total explained var 41\%)
  - residual \( \alpha_2 \rightarrow C_2 \quad 5\% \) (additional explained var 24\%)
Between Level Class Variables

- Between level heterogeneity

- \( C_{ij} = C_j \)

- How are between level class variables identified and interpreted?

- Is between level sample size important?
Between Level Class Variables - Simulation

\[ Y_{ij} = \mu_{cj} + \beta_{cj} X_{ij} + \varepsilon_{ij} \]

\[ \text{Var}(\mu_{cj}) = \text{Var}(\beta_{cj}) = v \]

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>( v )</th>
<th>MSE of ( \alpha_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>within</td>
<td>0</td>
<td>0.31</td>
</tr>
<tr>
<td>Model 2</td>
<td>between</td>
<td>0</td>
<td>0.10</td>
</tr>
<tr>
<td>Model 3</td>
<td>between</td>
<td>0.1</td>
<td>0.26</td>
</tr>
<tr>
<td>Model 4</td>
<td>between</td>
<td>0.2</td>
<td>0.44</td>
</tr>
</tbody>
</table>
Level Class Variables Status

Is Population Heterogeneity a Between Level or a Within Level Phenomenon? Modeling options are

- Within Latent Class Variable: $Var(\alpha_{cj}) = 0$
- Between Latent Class Variable: $Var(\alpha_{cj}) = \text{huge}$
- Within-Between Latent Class Variable: $Var(\alpha_{cj}) > 0$
### Level Class Variables Status - Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Between C</th>
<th>Within-Between C C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>1</td>
<td>0.90(0.14)</td>
<td>0.90(0.13)</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0</td>
<td>0.13(0.13)</td>
<td>0.12(0.12)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.2</td>
<td>0.25(0.07)</td>
<td>0.25(0.06)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.8</td>
<td>0.81(0.07)</td>
<td>0.81(0.06)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>1.01(0.04)</td>
<td>1.01(0.04)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2</td>
<td>0.22(0.08)</td>
<td>0.22(0.08)</td>
</tr>
<tr>
<td>$LL$</td>
<td></td>
<td>-1517.6</td>
<td>-1517.6</td>
</tr>
</tbody>
</table>

$$ICC(C)=\frac{Var(\alpha_1)}{Var(\alpha_1)+\pi^2/3}=0.995$$
The Grade of Membership Model

- Extension of LCA - Latent class variable for each item

- Partial class membership

- Individuals in transitional state: in between classes.

- Some measurements indicate one class, other measurements indicate another class
The Grade of Membership Model continued

- **LCA**

\[ P(Y_{ij} = 1 | C_j = c) = \Phi(\tau_{ic}) \]

\[ P(C_j = 1) = \frac{\exp(\alpha_1)}{1 + \exp(\alpha_1)} \]

- **GoM**

\[ P(Y_{ij} = 1 | C_{ij} = c) = \Phi(\tau_{ic}) \]

\[ P(C_{ij} = 1) = \frac{\exp(\alpha_{1j})}{1 + \exp(\alpha_{1j})} \]
The Grade of Membership Model Example

Antisocial Behavior (ASB) data from the National Longitudinal Survey of Youth (NLSY). 17 binary items: 8 property offense items, 5 personal offense items and 4 drug offense items.

<table>
<thead>
<tr>
<th>Model</th>
<th>2 class</th>
<th>3 class</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCA</td>
<td>-42625.7 (35)</td>
<td>-41713.1 (53)</td>
</tr>
<tr>
<td>GoM</td>
<td>-42159.1 (36)</td>
<td>-41554.6 (55)</td>
</tr>
</tbody>
</table>
The Grade of Membership Model Example

17 Items Probability Profiles for the Offense Prone Class for GoM v.s. LCA models. Correlation 99%.
Combining LCA, IRT and GoM Models: GoM-FMA

- FMA model

\[ P(Y_{ij} = 1 | C_j = c) = \Phi(\tau_{ic} + \lambda_{ic}\eta_j) \]

\[ P(C_j = 1) = \frac{\exp(\alpha_1)}{1 + \exp(\alpha_1)} \]

- GoM-FMA model

\[ P(Y_{ij} = 1 | C_{ij} = c) = \Phi(\tau_{ic} + \lambda_{ic}\eta_j) \]

\[ P(C_{ij} = 1) = \frac{\exp(\alpha_{1j})}{1 + \exp(\alpha_{1j})} \]
GoM-FMA Example

UCLA clinical sample of 425 males with ADHD diagnosis. The data consists of 9 inattentiveness items and 9 hyperactivity items, all dichotomous.

<table>
<thead>
<tr>
<th>2 Class Model</th>
<th>Log-Likelihood</th>
<th>Number of Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCA</td>
<td>-3650.0</td>
<td>37</td>
</tr>
<tr>
<td>FMA</td>
<td>-3502.4</td>
<td>56</td>
</tr>
<tr>
<td>FMA-GoM</td>
<td>-3501.7</td>
<td>57</td>
</tr>
</tbody>
</table>

ICC(C)=86%
Multilevel Mixture Models with Survey Data

- Sampling Weights on the Within and the Between Level
- Stratification
- Multistage Sampling: PSU and SSU
Multilevel Mixture Models with Non-Normal Data

- Binary, Polytomous, Nominal, Poisson, Censored, Two-Part

- Survival Analysis: Cox regression, Finite Mixtures of Frailty Models
Technical Aspects

- Random Starts
  - Starting values from simple models
  - Replicating the best solution

- Numerical Integration
  - Adaptive and Non-Adaptive Integration
  - Gauss-Hermite, Regular, Monte-Carlo Integration
  - Parametrization, Cholesky option
Conclusions

• Mplus 4.1

• Mplus 4.2
  - Multiple Class Variables for Two-Level Models
  - Between Level Class Variables
  - Multiple Within, Between and Within-Between Class Variables