Considering Alternative Metrics of Time: Does Anybody Really Know What “Time” Is?

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Road Map

- Steps in longitudinal analysis
- The missing step
- Alternative metrics of time
- What about time?
Longitudinal Designs...

• ...Have become ubiquitous across many disciplines
  ➢ Growth in scholastic achievement in children
  ➢ Improvement in job performance of employees
  ➢ Changes in marital satisfaction in spouses
  ➢ Physical and cognitive decline in older adults

• ... Are the only way to measure individual change
  ➢ Also (usually) offer benefits of cross-sectional studies, too
  ➢ **Between-Person** (BP), **INTER**-individual, cross-sectional variation
  ➢ **Within-Person** (WP), **INTRA**-individual, longitudinal variation

Goals of Longitudinal Models

• 5 rationales of longitudinal research
  ➢ Baltes & Nesselroade, 1979
    • Chapter 1 from *Longitudinal Research in the Study of Behavior and Development*

• 7 levels of longitudinal analysis
  ➢ Hofer & Sliwinski, 2006
    • Chapter 2 from *Handbook of the Psychology of Aging (6th edition)*

• 7+ steps in longitudinal modeling
  ➢ e.g., Singer & Willett, 2003
    • Chapter 4 from *Applied Longitudinal Data Analysis*
  ➢ Applicable to both MLM and SEM analytic frameworks
Longitudinal Analysis: Step 1

Do you even have longitudinal data?

• Calculate an **IntraClass Correlation** from an empty model:

  \[
  L1: \quad y_{ti} = \beta_{0i} + e_{ti} \\
  L2: \quad \beta_{0i} = \gamma_{00} + U_{0i}
  \]

  \[
  ICC = \frac{BP \text{ Variance}}{BP \text{ Variance} + WP \text{ Variance}} = \frac{\tau_{u0}^2}{\tau_{u0}^2 + \sigma_e^2}
  \]

• **ICC** = proportion of variance that is constant over time
  
  **ICC** = proportion of variance that is cross-sectional

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Longitudinal Analysis: Step 3

• What is the pattern of **average change** over time?

  ✓ Plot ML estimated means and individual trajectories
  ✓ What shape do they take?
    • Linear or nonlinear? Continuous or discontinuous?

<table>
<thead>
<tr>
<th>Parsimony</th>
<th>Empty Model: Predicts NO change over time</th>
<th>Most useful model: likely somewhere in between!</th>
<th>Saturated Means: Reproduces mean at each occasion</th>
<th>Good fit</th>
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<tbody>
<tr>
<td>1 Fixed Effect</td>
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<td># Fixed Effects = # Occasions</td>
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Longitudinal Analysis: Step 4

• Which aspects of change show **individual differences**?
  ➢ Individual differences in outcome level? Slopes for change?

- **Example: Covariance matrix**

- **Most useful model: likely somewhere in between!**

- **Unstructured (UN)**

- **Compound Symmetry (CS)**

• Equivalently: what is the covariance pattern over time?
  ➢ Constant, increasing, or decreasing (co)variance across lags?

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Steps 3 & 4

• Where should your **intercept** be (where is time = 0)?
  ➢ Where would you like a snap-shot of individual differences?
  ➢ Consider both data and research questions
  ➢ No wrong place for your intercept

- **Step 3**: Average pattern of change  ➔ Test **fixed** effects
- **Step 4**: Individual differences in change  ➔ Test **random** effects

• Proceed with your “best-fit” (or really, least wrong) **unconditional model for change**...
Steps 3 & 4: Families of Change Models

- Polynomial models \((linear, \ quadratic, \ cubic...\)
  - Continuous nonlinear trajectories
  - Common use, low data requirements

- Piecewise models \((2 \ or \ more \ distinct \ slopes)\)
  - Discontinuous trajectories for a known reason
  - Useful for event-based designs

- Latent basis models \((estimated \ differences \ between \ times)\)
  - Flexible yet parsimonious

- Really nonlinear models \((nonlinear \ in \ parameters)\)
  - E.g., exponential, power, logistic curves
  - Flexible but data-demanding

Longitudinal Analysis: Step 5

- **Predict individual differences** in level and change
  - Why do people need their own intercepts/asymptotes?
  - Why do people need their own slopes/curves/rates for change?

- Test time-invariant predictors to account for any individual differences in level and change
  - Does the treatment group improve more than the control group?
  - Do more educated persons have lower rates of cognitive decline?

- Can also test differences in amount of BP variability
  - Are boys more heterogeneous in growth of height than girls?
Longitudinal Analysis: Step 6

- Predict **intra-individual deviation** from change
  - Why are you off your line today?

- Test time-varying predictors to account for any remaining time-specific variation
  - Fluctuation about usual levels of stress, illness, resources...
  - However: Time-varying predictors usually contain both BP and WP information, and thus usually more than one effect

- Can also test differences in amount of WP variability
  - Do younger adults fluctuate more in mood than older adults?

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Step 7 and beyond...

- Examine **multivariate relationships** of interest
  - BP correlations among level and change
    - Do persons who start out higher on X start out higher on Y also?
    - Do persons who change more on X change more on Y also?
    - Factor analytic examinations; lead-lag associations
  - WP coupling after controlling for level and change
    - Do X and Y rise and fall together over time?
    - Factor analytic examinations; lead-lag associations
  - BP or WP relationships among measures of variability
    - Does increased variability in performance precede cognitive decline?

- Examine other kinds of **heterogeneity** (mixture models)
  - Can individual differences be described discretely instead?
Road Map

- Steps in longitudinal analysis
- The missing step
- Alternative metrics of time
- What about time?

The Missing Step 2

- **Summary across steps**: The goal of creating statistical models of change is to describe the overall pattern of and predict individual differences in change over time.

- These models employ an often unrecognized assumption that we know exactly what “time” should be.

- The missing Step 2 involves 2 related concerns:
  - What should “time” be?
  - What do we do when people differ in “time”?
  - Concerns apply to accelerated longitudinal designs
Accelerated Longitudinal Designs

Want to do a longitudinal study but just don’t have the time?

**Accelerate**: Model trajectories over a wider span of time than directly possible using only observed longitudinal information

Does anybody really know what *Time* is*?

- **First: What should “time” be?**
  - What is the causal process by which we are indexing change?
  - What do we do when multiple processes may be at work?
  - Relevant for merging different persons onto same time metric, but not a relevant distinction within-persons

- **Consider the previous examples...**
  - Growth in scholastic achievement in children
  - Improvement in job performance of employees
  - Changes in marital satisfaction in spouses
  - Physical and cognitive decline in older adults

*Title with thanks to Chicago*
Does anybody really care (about Time)?

• Second: What do we do when people differ in “time”?
  ➢ When does change begin? Where do we start counting from?
  ➢ What extra modeling steps are needed when such design short-cuts are taken to fully cover the target metric of time?
  ➢ That is, how should our models distinguish between-person effects of time from within-person effects of time?

• Possible consequences of getting “time” wrong:
  ➢ Fixed time trends that don’t describe any individuals
  ➢ Individual differences that are distorted in magnitude
  ➢ Predictive relationships that are artifactual

Road Map

• Steps in longitudinal analysis

• The missing step

• Alternative metrics of time

• What about time?
Example Data: *Octogenarian (Twin) Study of Aging*

- 173 persons (65% women)
  - Measured up to 5 occasions over 8 years
  - Known dates of birth and death
  - Estimated dates of dementia diagnosis (91 Alz., 50 Vas., 32 Mixed)

- Baseline time ranges:
  - Age 79 to 100 (M = 84, SD = 3)
  - -16 to 0 years from death (M = -6, SD = 4)
  - -12 to 18 years from diagnosis (M = 0, SD = 5)

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<tr>
<td>Dementia</td>
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<td>.52</td>
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- Cognition outcomes (each T-scored):
  - General: Mini-Mental Status Exam
  - Memory: Object Recall
  - Spatial Reasoning: Block Design

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Current Focus: The Missing Step 2

- **First: What should “time” be?**
  - Which method of clocking time best matches the causal process thought to be responsible for observed change?
  - How can alternative metrics of time provide different pictures of change (i.e., mean trends, individual differences)?

- **Second: Where should we start counting from?**
  - How do we set up our model to fully account for all of the possible BP and WP effects of given time metric?
  - **Steps 3+ logically follow from this point**

- **Data:** Real! (and some simulated data, time permitting)
Alternative Metrics of Time

- **Chronological Age as Time (47% BP)**
  > Individual differences are organized around the mean level for a given *distance from birth* and change with distance from birth

- **Years to Death as Time (24% BP)**
  > Individual differences are organized around the mean level for a given *distance from death* and change with distance from death

- **Years to Dementia Diagnosis as Time (70% BP)**
  > Individual differences are organized around the mean level for a given *distance from diagnosis* and change with distance from diagnosis

- **Years in Study as Time (0% BP)**
  > Individual differences are organized around the mean level for a given *distance from baseline* and change with distance from baseline

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General Cognition: MMSE

![Graphs showing MMSE scores over time for different metrics]
Memory: Object Recall

Spatial Reasoning: Block Design
First Option: Age-as-Time

Level 1: \( Y_{ti} = \beta_{0i} + \beta_{1i}(\text{Age}_{ti} - 84) + \beta_{2i}(\text{Age}_{ti} - 84)^2 + e_{ti} \)

Level 2 Equations (one per \( \beta \)):

- \( \beta_{0i} = \gamma_{00} + U_{0i} \uparrow \) predicted \( Y \) when age=84
- \( \beta_{1i} = \gamma_{10} + U_{1i} \uparrow \) rate of \( \Delta \) when age=84
- \( \beta_{2i} = \gamma_{20} + U_{2i} \uparrow \) \( \frac{1}{2} \) rate of \( \Delta \) in \( \Delta \) per year

First Option: Age-as-Time

• If people differ in initial age, tracking change as a function of age requires assuming age convergence
  ➢ Younger people and older people differ only by age
  ➢ Between-person, cross-sectional age effects are equivalent to within-person, longitudinal aging effects

• Age convergence is not likely to hold when
  ➢ Initial age range is large (47% BP here)
  ➢ Cohort differences and selection effects are large

• Is exactly the same problem as not separating WP effects from BP effects of ANY time-varying predictor
Can use a variant of grand-mean-centering to test convergence of BP and WP age effects empirically.

**Level 1 Age-Based:**

\[ Y_{ti} = \beta_{0i} + \beta_{1i} (\text{Age}_{ti} - 84) + \beta_{2i} (\text{Age}_{ti} - 84)^2 + e_{ti} \]

**Level 2 Equations:**

\[ \beta_{0i} = \gamma_{00} + \gamma_{01} (\text{Age}_{T1i} - 84) + U_{0i} \]
\[ \beta_{1i} = \gamma_{10} + \gamma_{11} (\text{Age}_{T1i} - 84) + U_{1i} \]
\[ \beta_{2i} = \gamma_{20} + \gamma_{21} (\text{Age}_{T1i} - 84) + U_{2i} \]

*AgeT1* → Incremental effect of cross-sectional age (cohort)

Use *age at baseline* (or birth year) instead of mean age to lessen bias from attrition-related missing data.

Significance → Non-convergence

It matters **WHEN** you were 84.

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**Age-Based Models of MMSE**

[Graphs showing age convergence and baseline cohorts]

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Age-Based Models of Object Recall

Age-Based Models of Spatial Reasoning
So if age is just a time-varying predictor...

- Because years to death and years to dementia diagnosis also have BP variation (24%, 70%), the same concerns about testing convergence apply to them too.

- **Years to death**
  - L1: YTdeath_{t1} + 7
  - L2: YTdeathT1_{t1} + 7

- **Years to diagnosis**
  - L1: YTdem_{t1} − 0
  - L2: YTdemT1_{t1} − 0

- If L2 effects are significant, then it matters WHEN you were 7 years from death (or at the point of diagnosis).

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Death-Based and Dementia-Based Models of MMSE

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Death-Based and Dementia-Based Models of Object Recall

Death-Based and Dementia-Based Models of Spatial Reasoning
Comparing Models by Fit...

The fit of alternative metrics of time to the data can be compared using their information criteria...

### ML AIC

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### ML BIC

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Comparing Models by Variances...

The fit of alternative metrics of time to the data can also be compared using their variance components...

### Residual Variance

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### Intercept Variance

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</table>
Comparing Models By Data...

Road Map

• Steps in longitudinal analysis

• The missing step

• Alternative metrics of time

• What about time?
What about Time as “Time”?

- When the accelerated time metrics do not show convergence of their BP and WP time effects, an alternative model specification may be more useful.

- **Time-in-study models** separate BP and WP effects:
  - Accelerated metric (age, death...) → Grand-mean-centering
  - Time-in-study version → Person/group-mean-centering

- Time-in-study models can be made equivalent to accelerated time metric models in their fixed effects, but not in their random effects (stay tuned).

Model Variants Using Age

**Level 1 Age-Based (Grand-MC):**
\[ Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + e_{ti} \]

**Level 1 Time-Based (Person/Group-MC):**
\[ Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - AgeT1_i) + e_{ti} \]

**Level 2 Equations (same):**
\[
\begin{align*}
\beta_{0i} &= y_{00} + y_{01}(AgeT1_i - 84) + U_{0i} \\
\beta_{1i} &= y_{10} + y_{11}(AgeT1_i - 84) + U_{1i} 
\end{align*}
\]

**Effects of AgeT1 per model:**
- **Age-Based:** Incremental effect of cross-sectional age (cohort)
- **Time-Based:** Total effect of cross-sectional age (cohort+time)
Model Variants Using Years to Death

**Level 1 Death-Based (Grand-MC):**
\[ Y_{ti} = \beta_{0i} + \beta_{1i}(YTdeath_{ti} + 7) + e_{ti} \]

**Level 1 Time-Based (Person/Group-MC):**
\[ Y_{ti} = \beta_{0i} + \beta_{1i}(YTdeath_{ti} - YTdeathT1_{i}) + e_{ti} \]

**Level 2 Equations (same):**
\[ \beta_{0i} = \gamma_{00} + \gamma_{01}(YTdeathT1_{i} + 7) + U_{0i} \]
\[ \beta_{1i} = \gamma_{10} + \gamma_{11}(YTdeathT1_{i} + 7) + U_{1i} \]

**Effects of YTdeathT1:**
- **Death-Based:** Incremental effect of YTdeath (cohort)
- **Time-Based:** Total effect of YTdeath (cohort+time)

---

### Time as “Time”

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<th>Both</th>
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Effect of Age Cohort on Intercept
(Fixed L1 Linear Age Slope)

Time-in-Study ≈ Person/Group-MC:
L1: \[ Y_{ti} = \beta_0 + \beta_1 (Ageti - Age_{T1i}) + e_{ti} \]
L2: \[ \beta_0i = \gamma_{00} + \gamma_{01} (Age_{T1i}) + U_{0i} \]
\[ \beta_1i = \gamma_{10} \]

\[ Y_{ti} = \gamma_{00} + \gamma_{01} (Age_{T1i}) + \gamma_{10} (Ageti - Age_{T1i}) + U_{0i} + e_{ti} \]

\[ Y_{ti} = \gamma_{00} + \gamma_{01} (Age_{T1i}) + \gamma_{10} (Ageti) + U_{0i} + e_{ti} \]

Age-Based ≈ Grand-MC:
L1: \[ Y_{ti} = \beta_0i + \beta_1i (Ageti) + e_{ti} \]
L2: \[ \beta_0i = \gamma_{00} + \gamma_{01} (Age_{T1i}) + U_{0i} \]
\[ \beta_1i = \gamma_{10} \]

\[ Y_{ti} = \gamma_{00} + \gamma_{01} (Age_{T1i}) + \gamma_{10} (Ageti) + U_{0i} + e_{ti} \]

Effect of Age Cohort on L1 Age Slope
(Fixed L1 Linear Age Slope)

Time-in-Study ≈ Person/Group-MC:

\[ Y_{ti} = \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10}(Age_{T1i}) + \gamma_{01}(Age_{T1i}) + \gamma_{11}(Age_{T1i})^2 + \gamma_{02}(Age_{T1i})(Age_{T1i}) \]

Age-Based ≈ Grand-MC:

\[ Y_{ti} = \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10}(Age_{T1i}) + \gamma_{01}(Age_{T1i}) + \gamma_{11}(Age_{T1i})^2 + \gamma_{02}(Age_{T1i})(Age_{T1i}) \]

Intercept: \[ Y_{00} = \gamma_{00} \]
WP Effect: \[ Y_{10} = \gamma_{10} \]
BP Effect: \[ Y_{01} = \gamma_{01} + \gamma_{10} \]
BP\(^2\) Effect: \[ Y_{02} = \gamma_{02} + \gamma_{11} \]
BP*WP: \[ Y_{11} = \gamma_{11} \]

Must also add Age\(^2\) to retain equivalent models
Add Fixed Quadratic L1 Age Slope (Fixed L1 Age Slopes)

Time-in-Study = Person/Group-MC:

\[
Y_{ti} = \gamma_{00} + U_{0i} + \gamma_{10}(Age_{ti} - AgeT1_{i}) + \gamma_{20}(Age_{ti} - AgeT1_{i})^2 + \gamma_{01}(AgeT1_{i}) + \gamma_{02}(AgeT1_{i})^2 + \gamma_{11}(Age_{ti} - AgeT1_{i})(AgeT1_{i})
\]

\[
Y_{ti} = \gamma_{00} + U_{0i} + \gamma_{10}(Age_{ti}) + \gamma_{20}(Age_{ti})^2 + (\gamma_{01} - \gamma_{10})(AgeT1_{i}) + (\gamma_{02} + \gamma_{20} - \gamma_{11})(AgeT1_{i})^2 + (\gamma_{11} - 2\gamma_{20})(Age_{ti})(AgeT1_{i})
\]

Age-Based = Grand-MC:

\[
Y_{ti} = \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10}(Age_{ti}) + \gamma_{20}(Age_{ti})^2 + \gamma_{01}'(AgeT1_{i}) + \gamma_{02}'(AgeT1_{i})^2 + \gamma_{11}'(Age_{ti})(AgeT1_{i})
\]

Intercept: \(\gamma_{00} = \gamma_{00}\)  BP Effect: \(\gamma_{01} = \gamma_{01}' + \gamma_{10}\)
WP Effect: \(\gamma_{10} = \gamma_{10}\)  BP\(^2\) Effect: \(\gamma_{02} = \gamma_{02}' + \gamma_{11}' + \gamma_{20}\)
WP\(^2\) Effect: \(\gamma_{20} = \gamma_{20}\)  BP*WP: \(\gamma_{11} = \gamma_{11}' + 2\gamma_{20}\)

Time-in-Study Models so far...

- WP change is based only on longitudinal information
- Are equivalent WP across alternative accelerated time metrics
- Because unique information from the alternative time metrics is really only available BP, it only shows up in the BP model
- Can (usually) be made equivalent in their fixed effects to models based in alternative accelerated time metrics
- So why bother? Random effects
Random Slopes across Models

**Time-in-Study = Person/Group-MC:**
\[ Y_{ti} = \gamma_{00} + \gamma_{01}(\text{Age}_{T1i}) + \gamma_{10}(\text{Age}_{ti} - \text{Age}_{T1i}) + U_{0i} + U_{1i}(\text{Age}_{ti} - \text{Age}_{T1i}) + e_{ti} \]

**Age-Based ≈ Grand-MC:**
\[ Y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(\text{Age}_{T1i}) + \gamma_{10}(\text{Age}_{ti}) + U_{0i} + U_{1i}(\text{Age}_{ti} - \text{Age}_{T1i}) + e_{ti} \]

Both centerings yield equivalent models if the L1 age slope is fixed, but NOT if the slope is random.

**AgeT1 is NOT subtracted out of the random slope in Age-based Grand-MC. Therefore, these models with random slopes will not be equivalent.**

So which do we choose?

Random Effects Across Models

- **Random intercepts** mean different things under each model:
  - **Person-MC** → Individual differences at time=0 (everyone has)
  - **Grand-MC** → Individual differences at age=0 (not everyone has)

- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
  - **Person-MC** → Won’t affect shrinkage of slopes unless highly correlated
  - **Grand-MC** → Will affect shrinkage of slopes due to forced extrapolation

- As a result, the **random slope variance may be smaller** under grand-MC (age, death...) than under person-MC (time)
  - Problem worsens with greater BP variation in time (more extrapolation)
  - Anecdotal example using clustered data was presented in Raudenbush & Bryk (2002; chapter 6)
Bias in Random Age Slope Variance

Top: Intercepts & slopes are homogenized in grand-MC
Right: Bias in random slope variance under grand-MC

Slope Variance in Example Models

- Slope variance estimate was indeed **33-77% larger** in the time-based model versions across outcomes...

Years-Since-Birth (47% BP)  
Years-to-Death (24% BP)
... Although model fit was the same

Simulation Study Results
(Generated by Time, Analyzed by Age)

Percent Bias in Random Slope Variance

- ICC Age = 0.2
- ICC Age = 0.5
- ICC Age = 0.8

June 2010
And so the winner is... TIME?

- Although seemingly the most non-informative choice, simply tracking change as a function of study duration:
  - Represents WP changes as directly and parsimoniously as possible
  - Seems to recover change slope variance better
  - Permits inclusion of persons who have not experienced events in an informative time metric (death, dementia diagnosis)
    - Piecewise models can include differential change before/after event

- Because time-in-study models make no assumptions about the processes causing change, these become testable hypotheses
  - Do persons who are older decline faster?
    - Age*Time interaction
  - After considering mortality, do older persons still decline faster?
    - Competing Y|death*Time and Age*Time interactions

Conclusions

- The steps in conducting a longitudinal analysis should always carefully consider what “time” could and should be
  - Multiple processes may be at play simultaneously

- Given both BP and WP variation in time, modeling decisions can have important implications for the resulting inferences about pattern of change and individual differences therein
  - Carefully evaluate how to best account for BP differences
  - Otherwise, aggregate trends may not apply to individuals

- Such preliminary considerations are important pre-cursors to making informed use of advances in longitudinal modeling
Thank you!

Questions or comments?
Email me: Lhoffman2@unl.edu
Does anybody really care (about \textit{Time})?

- Even in longitudinal studies focused on \textbf{within-person fluctuation rather than change}, time may still be relevant.

- For instance, in daily diary studies:
  - Day of the Week (time metric could be \textit{day of week})
  - Fatigue/Reactivity (time metric could be \textit{day of study})

- In these cases you’d be “controlling for change” instead of “modeling change” (same models, different emphasis)
  - Some examples...

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Plans to Drink Alcohol by “Time”

- #Drinks by Interview Week Number
- #Drinks by Time to Spring Break

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Change in Negative Affect over “Time”

Stawski & Sliwinski, 2005

Aging
.24/burst (6 mos.)
$p < .0001$

Reactivity
-.07/session
$p < .01$