Considering Alternative Metrics of Time: Does Anybody Really Know What "Time" Is?

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Road Map

- Steps in longitudinal analysis
- The missing step
- Alternative metrics of time
- What about time?

Longitudinal Designs...

- ...Have become ubiquitous across many disciplines
 - > Growth in scholastic achievement in children
 - > Improvement in job performance of employees
 - > Changes in marital satisfaction in spouses
 - > Physical and cognitive decline in older adults
- ... Are the only way to measure individual change
 - > Also (usually) offer benefits of cross-sectional studies, too
 - > Between-Person (BP), INTER-individual, cross-sectional variation
 - > Within-Person (WP), INTRA-individual, longitudinal variation

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Goals of Longitudinal Models

- 5 rationales of longitudinal research
 - > Baltes & Nesselroade, 1979
 - Chapter 1 from Longitudinal Research in the Study of Behavior and Development
- 7 levels of longitudinal analysis
 - > Hofer & Sliwinski, 2006
 - Chapter 2 from Handbook of the Psychology of Aging (6th edition)
- 7+ steps in longitudinal modeling
 - > e.g., Singer & Willett, 2003
 - Chapter 4 from Applied Longitudinal Data Analysis
 - > Applicable to both MLM and SEM analytic frameworks

Longitudinal Analysis: Step 1

Do you even have longitudinal data?

Calculate an IntraClass Correlation from an empty model:

L1:
$$y_{ti} = \beta_{0i} + e_{ti}$$

L2: $\beta_{0i} = \gamma_{00} + U_{0i}$

ICC =
$$\frac{\text{BP Variance}}{\text{BP Variance} + \text{WP Variance}} = \frac{\tau_{\text{U}_0}^2}{\tau_{\text{U}_0}^2 + \sigma_{\text{e}}^2}$$

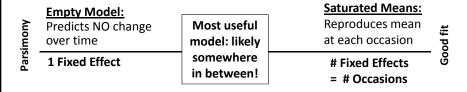
ICC = proportion of variance that is constant over time
 ICC = proportion of variance that is cross-sectional

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Longitudinal Analysis: Step 3

- What is the pattern of average change over time?
 - > Plot ML estimated means and individual trajectories
 - > What shape do they take?
 - Linear or nonlinear? Continuous or discontinuous?



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Longitudinal Analysis: Step 4

- Which aspects of change show individual differences?
 - > Individual differences in outcome level? Slopes for change?



- Equivalently: what is the covariance pattern over time?
 - > Constant, increasing, or decreasing (co)variance across lags?

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Steps 3 & 4

- Where should your **intercept** be (where is time = 0)?
 - > Where would you like a snap-shot of individual differences?
 - > Consider both data and research questions
 - > No wrong place for your intercept
- Step 3: Average pattern of change → Test fixed effects
- Step 4: Individual differences in change → Test random effects
- Proceed with your "best-fit" (or really, least wrong)
 unconditional model for change...

Steps 3 & 4: Families of Change Models

- Polynomial models (linear, quadratic, cubic...)
 - > Continuous nonlinear trajectories
 - > Common use, low data requirements
- Piecewise models (2 or more distinct slopes)
 - > Discontinuous trajectories for a known reason
 - > Useful for event-based designs
- Latent basis models (estimated differences between times)
 - > Flexible yet parsimonious
- Really nonlinear models (nonlinear in parameters)
 - > E.g., exponential, power, logistic curves
 - > Flexible but data-demanding

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Longitudinal Analysis: Step 5

- Predict individual differences in level and change
 - > Why do people need their own intercepts/asymptotes?
 - > Why do people need their own slopes/curves/rates for change?
- Test time-invariant predictors to account for any individual differences in level and change
 - > Does the treatment group improve more than the control group?
 - > Do more educated persons have lower rates of cognitive decline?
- Can also test differences in amount of BP variability
 - > Are boys more heterogeneous in growth of height than girls?

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Longitudinal Analysis: Step 6

- Predict intra-individual deviation from change
 - > Why are you off your line today?
- Test time-varying predictors to account for any remaining time-specific variation
 - > Fluctuation about usual levels of stress, illness, resources...
 - > However: Time-varying predictors usually contain both BP and WP information, and thus usually more than one effect
- Can also test differences in amount of WP variability
 - > Do younger adults fluctuate more in mood than older adults?

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Step 7 and beyond...

- Examine multivariate relationships of interest
 - > BP correlations among level and change
 - Do persons who start out higher on X start out higher on Y also?
 - Do persons who change more on X change more on Y also?
 - Factor analytic examinations; lead-lag associations
 - > WP coupling after controlling for level and change
 - Do X and Y rise and fall together over time?
 - Factor analytic examinations; lead-lag associations
 - > BP or WP relationships among measures of variability
 - Does increased variability in performance precede cognitive decline?
- Examine other kinds of heterogeneity (mixture models)
 - > Can individual differences be described discretely instead?

Road Map

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- · The missing step
- · Alternative metrics of time
- What about time?

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The Missing Step 2

- **Summary across steps**: The goal of creating statistical models of change is to describe the overall pattern of and predict individual differences in **change over time**.
- These models employ an often unrecognized assumption that we know exactly what "time" should be.
- The missing Step 2 involves 2 related concerns:
 - > What should "time" be?
 - > What do we do when people differ in "time"?
 - > Concerns apply to accelerated longitudinal designs

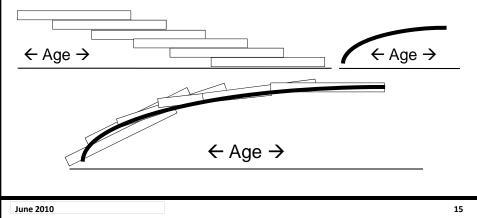
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Accelerated Longitudinal Designs

Want to do a longitudinal study but just don't have the time?

Accelerate: Model trajectories over a wider span of time than directly possible using only observed longitudinal information



Does anybody really know what *Time* is*?

- First: What should "time" be?
 - > What is the **causal process** by which we are indexing change?
 - > What do we do when multiple processes may be at work?
 - > Relevant for merging different persons onto same time metric, but not a relevant distinction within-persons
- Consider the previous examples...
 - > Growth in scholastic achievement in children
 - > Improvement in job performance of employees
 - > Changes in marital satisfaction in spouses
 - > Physical and cognitive decline in older adults

^{*} Title with thanks to Chicago

Does anybody really care (about *Time*)?

- Second: What do we do when people differ in "time"?
 - > When does change begin? Where do we start counting from?
 - > What extra modeling steps are needed when such design short-cuts are taken to fully cover the target metric of time?
 - > That is, how should our models distinguish *between-person* effects of time from *within-person* effects of time?
- Possible consequences of getting "time" wrong:
 - > Fixed time trends that don't describe any individuals
 - > Individual differences that are distorted in magnitude
 - > Predictive relationships that are artifactual

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Example Data: Octogenarian (Twin) Study of Aging

- 173 persons (65% women)
 - > Measured up to 5 occasions over 8 years
 - > Known dates of birth and death
 - > Estimated dates of dementia diagnosis (91 Alz., 50 Vas., 32 Mixed)
- Baseline time ranges:
 - \rightarrow Age 79 to 100 (M = 84, SD = 3)
 - \rightarrow -16 to 0 years from death (M = -6, SD = 4)
 - \rightarrow -12 to 18 years from diagnosis (M = 0, SD = 5)

Correlation	Age	Death
Death	.23	
Dementia	.17	.52

- Cognition outcomes (each T-scored):
 - > General: Mini-Mental Status Exam
 - > Memory: Object Recall
 - > Spatial Reasoning: Block Design

1	2	3	4	5
28	37	36	36	35
29	31	39	29	18
37	32	31	22	19

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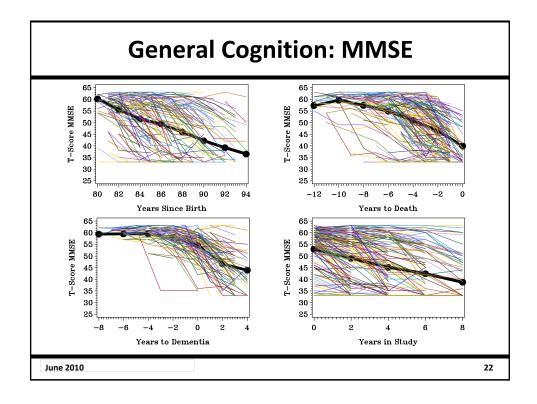
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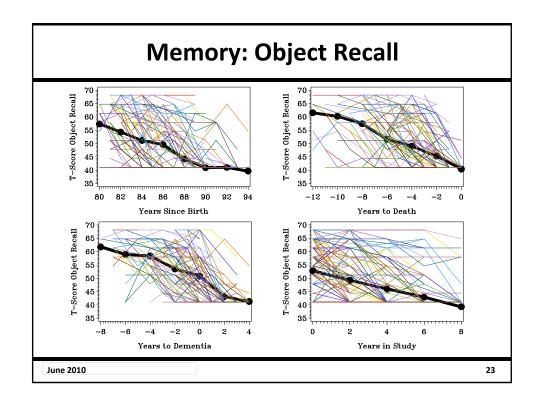
Current Focus: The Missing Step 2

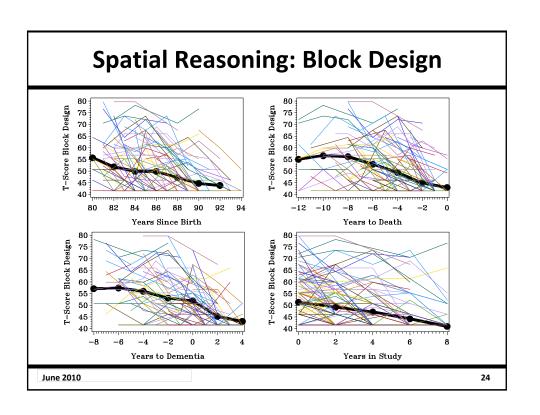
- · First: What should "time" be?
 - > Which method of clocking time best matches the causal process thought to be responsible for observed change?
 - > How can alternative metrics of time provide different pictures of change (i.e., mean trends, individual differences)?
- Second: Where should we start counting from?
 - > How do we set up our model to fully account for all of the possible BP and WP effects of given time metric?
 - > Steps 3+ logically follow from this point
- Data: Real! (and some simulated data, time permitting)

Alternative Metrics of Time

- Chronological Age as Time (47% BP)
 - > Individual differences are organized around the mean level for a given distance from birth and change with distance from birth
- Years to Death as Time (24% BP)
 - > Individual differences are organized around the mean level for a given distance from death and change with distance from death
- Years to Dementia Diagnosis as Time (70% BP)
 - > Individual differences are organized around the mean level for a given distance from diagnosis and change with distance from diagnosis
- Years in Study as Time (0% BP)
 - > Individual differences are organized around the mean level for a given distance from baseline and change with distance from baseline







First Option: Age-as-Time

Level 1:
$$Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + \beta_{2i}(Age_{ti} - 84)^2 + e_{ti}$$

Level 2 Equations (one per β):

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First Option: Age-as-Time

- If people differ in initial age, tracking change as a function of age requires assuming **age convergence**
 - > Younger people and older people differ only by age
 - > Between-person, cross-sectional *age effects* are equivalent to within-person, longitudinal *aging effects*
- Age convergence is not likely to hold when
 - > Initial age range is large (47% BP here)
 - > Cohort differences and selection effects are large
- Is exactly the same problem as not separating WP effects from BP effects of ANY time-varying predictor

Examining Age Convergence Effects

Can use a variant of **grand-mean-centering** to test convergence of BP and WP age effects empirically

Level 1 Age-Based:

$$Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + \beta_{2i}(Age_{ti} - 84)^2 + e_{ti}$$

Level 2 Equations:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(AgeT1_i - 84) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(AgeT1_i - 84) + U_{1i}$$

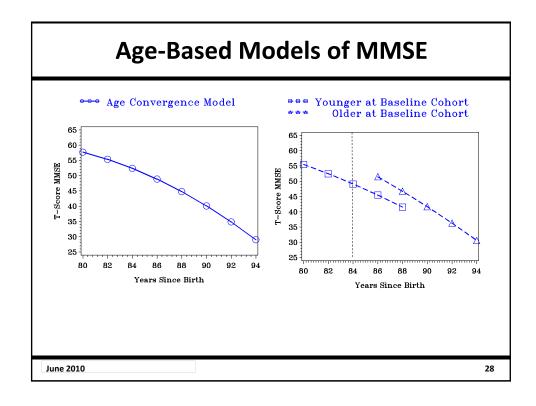
$$\beta_{2i} = \gamma_{20} + \gamma_{21}(AgeT1_i - 84) + U_{2i}$$

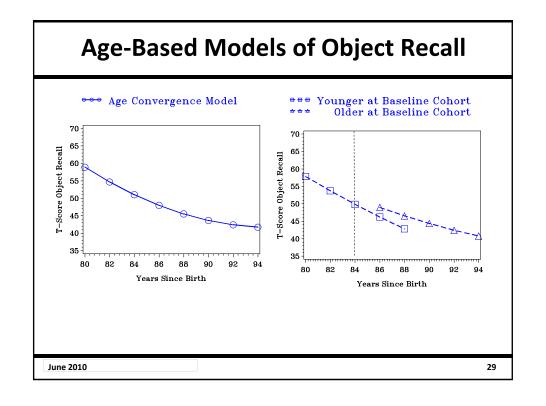
AgeT1 → Incremental effect of cross-sectional age (**cohort**)

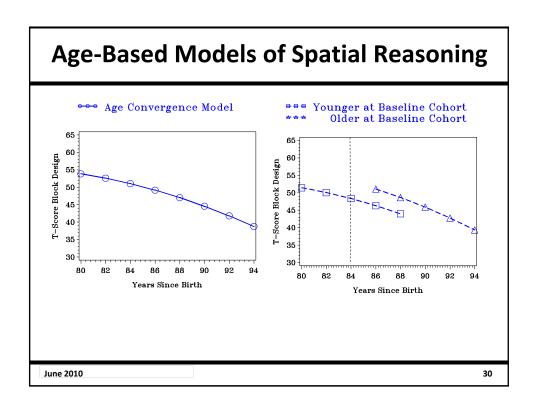
Use **age at baseline** (or birth year) instead of mean age to lessen bias from attrtion-related missing data

Significance → Non-convergence It matters WHEN you were 84

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So if age is just a time-varying predictor...

- Because years to death and years to dementia diagnosis also have BP variation (24%, 70%), the same concerns about testing convergence apply to them too
- Years to death

> L1: YTdeath_{ti} + 7

L2: YTdeathT1_i + 7

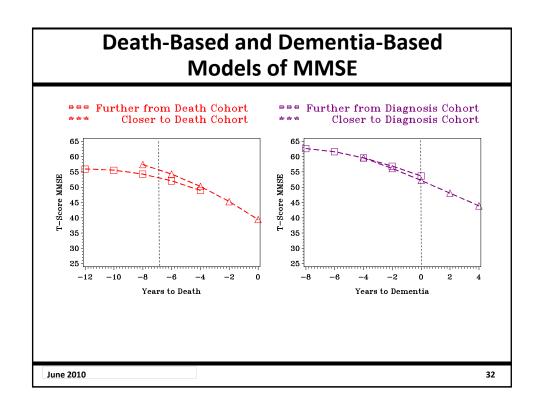
Years to diagnosis

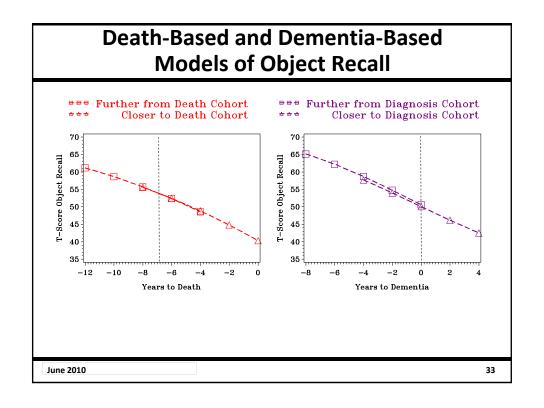
▶ L1: YTdem_{ti} – 0

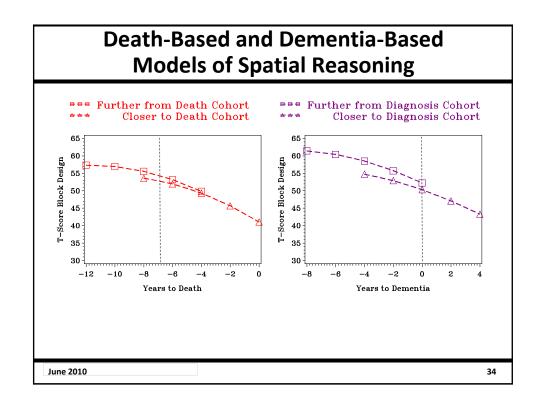
▶ L2: YTdemT1_i – 0

• If L2 effects are significant, then it matters WHEN you were 7 years from death (or at the point of diagnosis)

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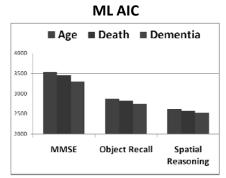


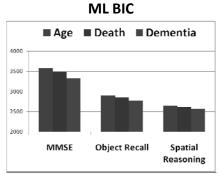




Comparing Models by Fit...

The fit of alternative metrics of time to the data can be compared using their information criteria...



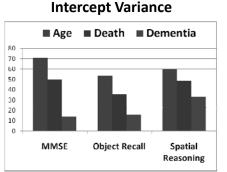


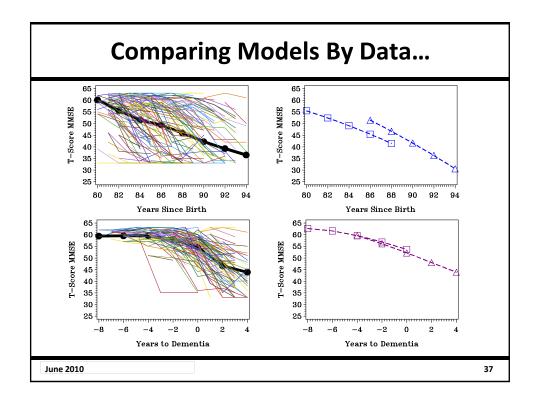
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Comparing Models by Variances...

The fit of alternative metrics of time to the data can also be compared using their variance components...

Residual Variance Age Death Dementia Object Recall Spatial Reasoning





Road Map

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What about Time as "Time"?

- When the accelerated time metrics do not show convergence of their BP and WP time effects, an alternative model specification may be more useful
- Time-in-study models separate BP and WP effects
 - ➤ Accelerated metric (age, death...) → Grand-mean-centering
 - ➤ Time-in-study version → Person/group-mean-centering
- Time-in-study models can be made equivalent to accelerated time metric models in their fixed effects, but not in their random effects (stay tuned)

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Model Variants Using Age

Level 1 Age-Based (Grand-MC):

$$Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + e_{ti}$$

<u>Level 1 Time-Based (Person/Group-MC)</u>:

$$Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - AgeT1_i) + e_{ti}$$

Level 2 Equations (same):

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(AgeT1_i - 84) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(AgeT1_i - 84) + U_{1i}$$

Effects of AgeT1 per model:

Age-Based: Incremental effect of cross-sectional age (cohort)

Time-Based: Total effect of cross-sectional age (cohort+time)

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Model Variants Using Years to Death

Level 1 Death-Based (Grand-MC):

 $Y_{ti} = \beta_{0i} + \beta_{1i}(YTdeath_{ti} + 7) + e_{ti}$

Level 1 Time-Based (Person/Group-MC):

 $Y_{ti} = \beta_{0i} + \beta_{1i}(YTdeath_{ti} - YTdeathT1_i) + e_{ti}$

Level 2 Equations (same):

Effects of YTdeathT1:

 $\beta_{0i} = \gamma_{00} + \gamma_{01} (YTdeathT1_i + 7) + U_{0i}$

Death-Based: Incremental effect of YTdeath (cohort)

 $\beta_{1i} = \gamma_{10} + \gamma_{11} (YTdeathT1_i + 7) + U_{1i}$

Time-Based: Total effect of YTdeath (cohort+time)

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Time as "Time"

Info:	ВР	Both	WP	ВР	Both	WP
Years in Study	AgeT1 _i	Age _{ti}	Age _{ti} – AgeT1 _i	YTdeathT1 _i	YTdeath _{ti}	YTdeath _{ti} – YTdeathT1 _i
0	80	80	0	-12	-12	0
2	80	82	2	-12	-10	2
4	80	84	4	-12	-8	4
0	80	80	0	-8	-8	0
2	80	82	2	-8	-6	2
4	80	84	4	-8	-4	4
0	84	84	0	-12	-12	0
2	84	86	2	-12	-10	2
4	84	88	4	-12	-8	4
0	84	84	0	-8	-8	0
2	84	86	2	-8	-6	2
4	84	88	4	-8	-4	4

Effect of Age Cohort on Intercept (Fixed L1 Linear Age Slope)

<u>Time-in-Study</u> ≈ <u>Person/Group-MC</u>:

L1:
$$Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - AgeT1_i) + e_{ti}$$

L2:
$$\beta_{0i} = \gamma_{00} + \gamma_{01}(AgeT1_i) + U_{0i}$$

$$\beta_{1i} = \gamma_{10}$$

$$\rightarrow Y_{ti} = \gamma_{00} + \gamma_{01}(AgeT1_i) + \gamma_{10}(Age_{ti} - AgeT1_i) + U_{0i} + e_{ti}$$

$$\rightarrow$$
 Y_{ti} = γ_{00} + (γ_{01} - γ_{10})(AgeT1_i) + γ_{10} (Age_{ti}) + U_{0i} + e_{ti}

←In terms of Time
←In terms of Age

Age-Based ≈ Grand-MC:

L1:
$$Y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti}) + e_{ti}$$

L2:
$$\beta_{0i} = \gamma_{00} + \gamma_{01}*(AgeT1_i) + U_{0i}$$

$$\beta_{1i} = \gamma_{10}$$

$$Y_{ti} = \gamma_{00} + \gamma_{01}*(AgeT1_i) + \gamma_{10}(Age_{ti}) + U_{0i} + e_{ti}$$

Term	P-MC	G-MC
Intercept	Υ ₀₀	Y ₀₀
WP Effect	Y ₁₀	Y ₁₀
Context	$\gamma_{01} - \gamma_{10}$	Y ₀₁ *
BP Effect	Y ₀₁	Y ₀₁ *+ Y ₁₀

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Effect of Age Cohort on L1 Age Slope (Fixed L1 Linear Age Slope)

<u>Time-in-Study</u> ≈ <u>Person/Group-MC</u>:

$$\begin{aligned} Y_{ti} &= \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10} (Age_{ti} - AgeT1_i) + \gamma_{01} (AgeT1_i) \\ &+ \gamma_{02} (AgeT1_i)^2 + \gamma_{11} (Age_{ti} - AgeT1_i) (AgeT1_i) \end{aligned}$$

$$Y_{ti} = \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10}(Age_{ti}) + (\gamma_{01} - \gamma_{10})(AgeT1_i) + (\gamma_{02} - \gamma_{11})(AgeT1_i)^2 + \gamma_{11}(Age_{ti})(AgeT1_i)$$

←In terms of Time
←In terms of Age

Age-Based ≈ Grand-MC:

$$Y_{ti} = \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10}(Age_{ti}) + \gamma_{01}^*(AgeT1_i) + \gamma_{02}^*(AgeT1_i)^2 + \gamma_{11}(Age_{ti})(AgeT1_i)$$

Must also add AgeT1² to retain equivalent models

Intercept: $\gamma_{00} = \gamma_{00}$ BP Effect: $\gamma_{01} = \gamma_{01}^* + \gamma_{10}$ WP Effect: $\gamma_{10} = \gamma_{10}$ BP² Effect: $\gamma_{02} = \gamma_{02}^* + \gamma_{11}$

BP*WP: $\gamma_{11} = \gamma_{11}$

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Add Fixed Quadratic L1 Age Slope (Fixed L1 Age Slopes)

Time-in-Study ≈ Person/Group-MC:

$$\begin{split} Y_{ti} &= \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10} (Age_{ti} - AgeT1_i) + \gamma_{20} (Age_{ti} - AgeT1_i)^2 \\ &+ \gamma_{01} (AgeT1_i) + \gamma_{02} (AgeT1_i)^2 + \gamma_{11} (Age_{ti} - AgeT1_i) (AgeT1_i) \\ Y_{ti} &= \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10} (Age_{ti}) + \gamma_{20} (Age_{ti})^2 + (\gamma_{01} - \gamma_{10}) (AgeT1_i) \\ &+ (\gamma_{02} + \gamma_{20} - \gamma_{11}) (AgeT1_i)^2 + (\gamma_{11} - 2\gamma_{20}) (Age_{ti}) (AgeT1_i) \end{split}$$

Age-Based ≈ Grand-MC:

$$\begin{split} Y_{ti} &= \gamma_{00} + U_{0i} + e_{ti} + \gamma_{10} (Age_{ti}) + \gamma_{20} (Age_{ti})^2 + \gamma_{01}^* (AgeT1_i) \\ &+ \gamma_{02}^* (AgeT1_i)^2 + \gamma_{11}^* (Age_{ti}) (AgeT1_i) \end{split}$$

BP Effect: $\gamma_{01} = \gamma_{01}^* + \gamma_{10}$ Intercept: $\gamma_{00} = \gamma_{00}$ WP Effect: $\gamma_{10} = \gamma_{10}$ BP² Effect: $\gamma_{02} = \gamma_{02}^* + \gamma_{11}^* + \gamma_{20}^*$ BP*WP: $\gamma_{11} = \gamma_{11}^{*} + 2\gamma_{20}$

WP² Effect: $\gamma_{20} = \gamma_{20}$

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Time-in-Study Models so far...

- WP change is based only on longitudinal information
- Are equivalent WP across alternative accelerated time metrics
- Because unique information from the alternative time metrics is really only available BP, it only shows up in the BP model
- Can (usually) be made equivalent in their fixed effects to models based in alternative accelerated time metrics
- · So why bother? Random effects

Random Slopes across Models

<u>Time-in-Study</u> ≈ <u>Person/Group-MC</u>:

$$Y_{ti} = \gamma_{00} + \gamma_{01}(AgeT1_i) + \gamma_{10}(Age_{ti} - AgeT1_i) + U_{0i} + U_{1i}(Age_{ti} - AgeT1_i) + e_{ti}$$

$$Y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(AgeT1_{i}) + \gamma_{10}(Age_{ti}) + U_{0i} + U_{1i}(Age_{ti}) - U_{1i}(AgeT1_{i}) + e_{ti}$$

Age-Based ≈ Grand-MC:

$$Y_{ti} = \gamma_{00} + \gamma^*_{01}(AgeT1_i) + \gamma_{10}(Age_{ti}) + U_{0i} + U_{1i}(Age_{ti}) + e_{ti}$$

So which do we choose?

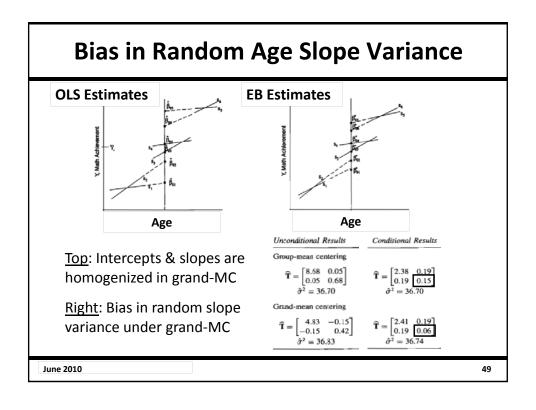
Both centerings yield equivalent models if the L1 age slope is fixed, but NOT if the slope is random.

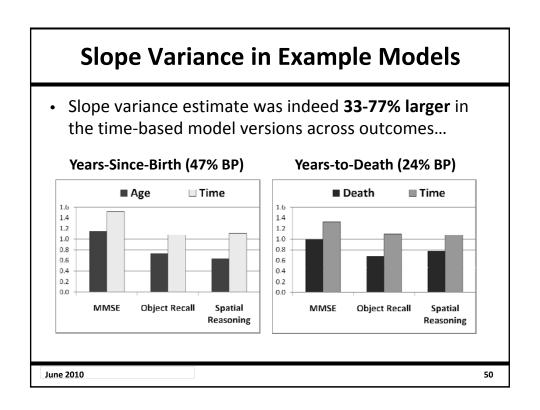
AgeT1 is NOT subtracted out of the random slope in Age-based Grand-MC. Therefore, these models with random slopes will not be equivalent.

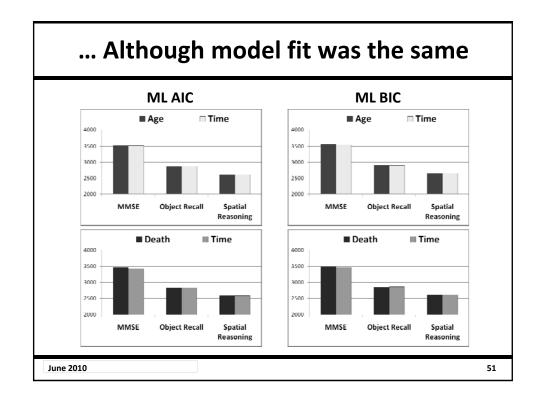
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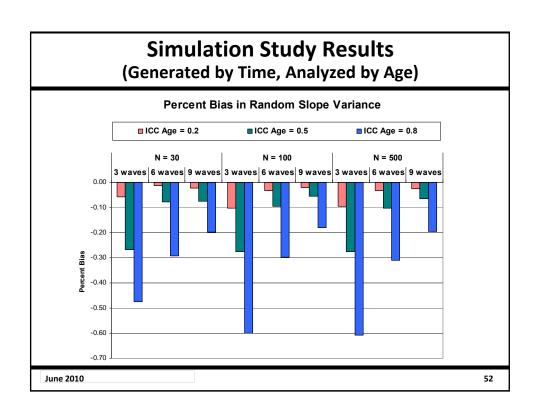
Random Effects Across Models

- Random intercepts mean different things under each model:
 - > Person-MC → Individual differences at time=0 (everyone has)
 - ➤ Grand-MC → Individual differences at age=0 (not everyone has)
- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
 - ▶ Person-MC → Won't affect shrinkage of slopes unless highly correlated
 - ➤ Grand-MC → Will affect shrinkage of slopes due to forced extrapolation
- As a result, the random slope variance may be smaller under grand-MC (age, death...) than under person-MC (time)
 - > Problem worsens with greater BP variation in time (more extrapolation)
 - Anecdotal example using clustered data was presented in Raudenbush & Bryk (2002; chapter 6)









And so the winner is... TIME?

- Although seemingly the most non-informative choice, simply tracking change as a function of study duration:
 - > Represents WP changes as directly and parsimoniously as possible
 - > Seems to recover change slope variance better
 - > Permits inclusion of persons who have not experienced events in an informative time metric (death, dementia diagnosis)
 - Piecewise models can include differential change before/after event
- Because time-in-study models make no assumptions about the processes causing change, these become testable hypotheses
 - > Do persons who are older decline faster?
 - Age*Time interaction
 - > After considering mortality, do older persons still decline faster?
 - Competing YTdeath*Time and Age*Time interactions

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Conclusions

- The steps in conducting a longitudinal analysis should always carefully consider what "time" could and should be
 - > Multiple processes may be at play simultaneously
- Given both BP and WP variation in time, modeling decisions can have important implications for the resulting inferences about pattern of change and individual differences therein
 - > Carefully evaluate how to best account for BP differences
 - > Otherwise, aggregate trends may not apply to individuals
- Such preliminary considerations are important pre-cursors to making informed use of advances in longitudinal modeling



Does anybody really care (about *Time*)?

- Even in longitudinal studies focused on within-person fluctuation rather than change, time may still be relevant
- · For instance, in daily diary studies:
 - > Day of the Week (time metric could be day of week)
 - > Fatigue/Reactivity (time metric could be day of study)
- In these cases you'd be "controlling for change" instead of "modeling change" (same models, different emphasis)
 - > Some examples...

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